A General Method for Calibrating Stochastic Channel Models with Kernels

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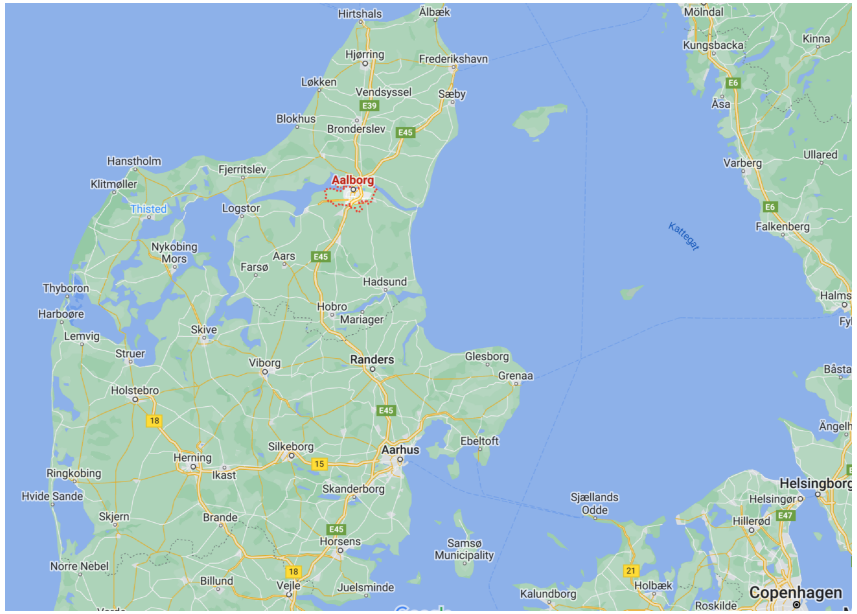
Introduction

Background

- B.E. in Electrical and Electronics engineering, BITS Pilani, India, 2015.
- M.Sc. in Signal processing and computing, Aalborg University, Denmark, 2017.
- PhD in Wireless Communications, Aalborg University, Denmark, 2021.

Current lab

- Probabilistic Machine Learning group at Department of Computer Science, Aalto Univeristy, Finland.
- Affiliated with FCAI.

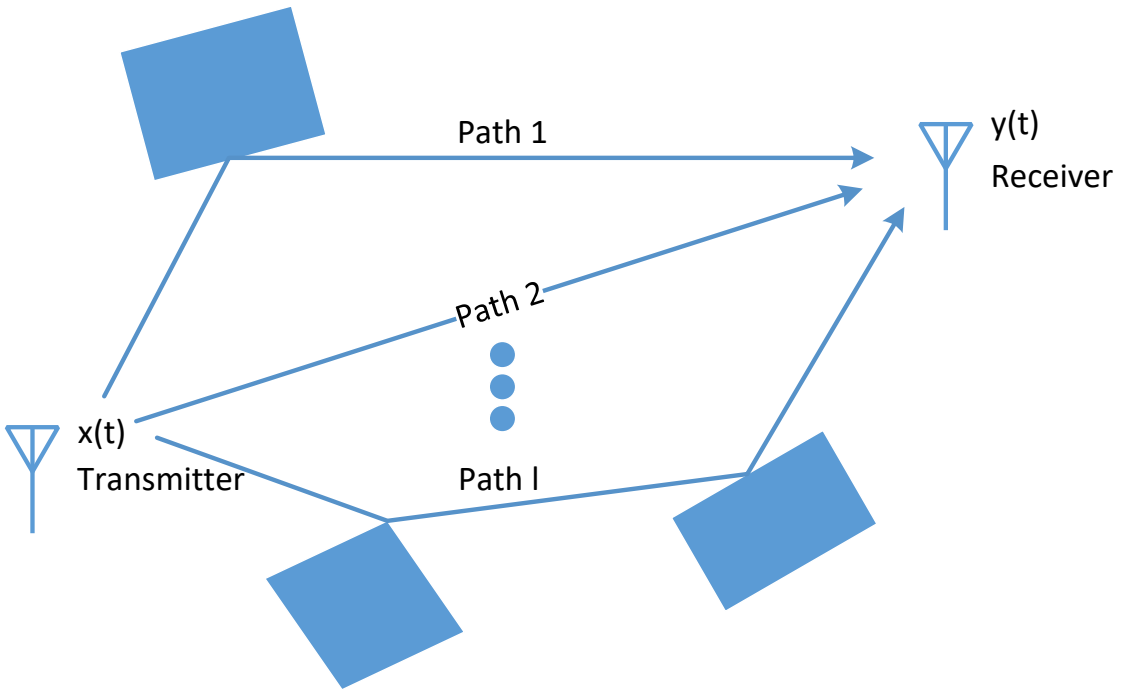


Outline

Radio Channel

The radio signal propagates from the transmitter to the receiver through the environment, termed the *radio channel*.

The communication systems engineers account for the radio channel using channel models.



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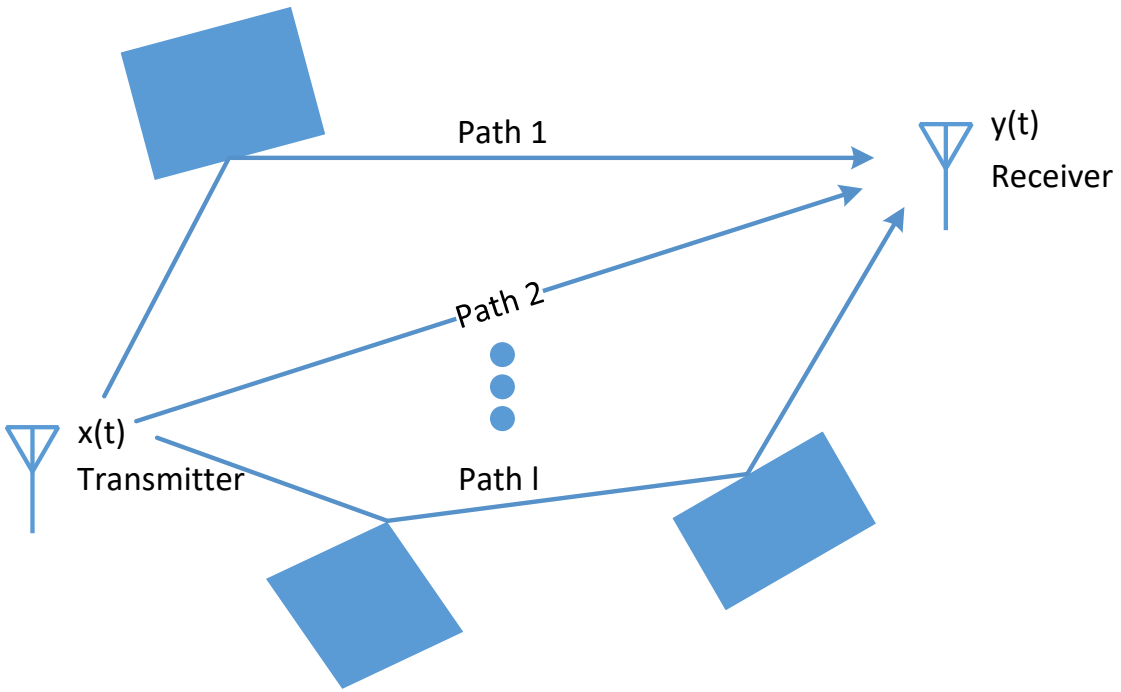

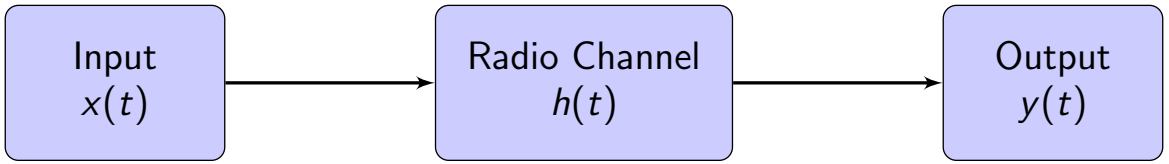

Quantity of interest: Impulse response $h(t)$ or transfer function $H(f)$

Stochastic Radio Channel Models

Consider the stochastic multipath model (assume time-invariance):

$$y(t) = (h * x)(t) + w(t) = \sum_{l} \beta_l x(t - \tau_l) + w(t)$$

- $h(t) \rightarrow$ channel impulse response
- $\{(\tau_l, \beta_l)\} \rightarrow$ delay and complex gain of lth multipath component
- $w(t) \rightarrow$ additive white complex Gaussian noise

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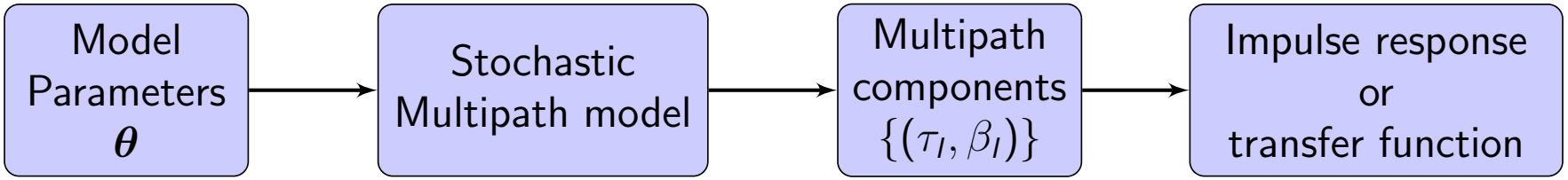
Simulating transfer function from a stochastic multipath model:



Model parameters $\boldsymbol{\theta}$ can be adjusted to mimic different environments.

Hence, their parameters need to be calibrated for the model to be useful.

Example: Model by Saleh-Valenzuela (S-V)

[Saleh and Valenzuela, 1987]

Multipaths arrive in clusters!

Impulse response:

$$h(t) = \sum_{l} \sum_{p} \beta_{pl} \delta \left(t - \left(T_l + \tau_{pl} \right) \right)$$

- $T_l \in \mathbb{R}^+$: cluster delay
- $\tau_{pl} \in \mathbb{R}^+$: within cluster delay
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Stochastic Model:

- $T_l \sim \mathrm{PoissonPP}(\mathbb{R}^+, \Lambda)$
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- $\beta_{pl} | T_l, \tau_{pl} \overset{iid}{\sim} \mathcal{CN}\left(0, Q \exp(-T_l/\Gamma) \exp(-\tau_{pl}/\gamma)\right)$

Parameters to be calibrated: $\boldsymbol{\theta} = [Q, \Lambda, \lambda, \Gamma, \gamma, \sigma_W^2]^\top$

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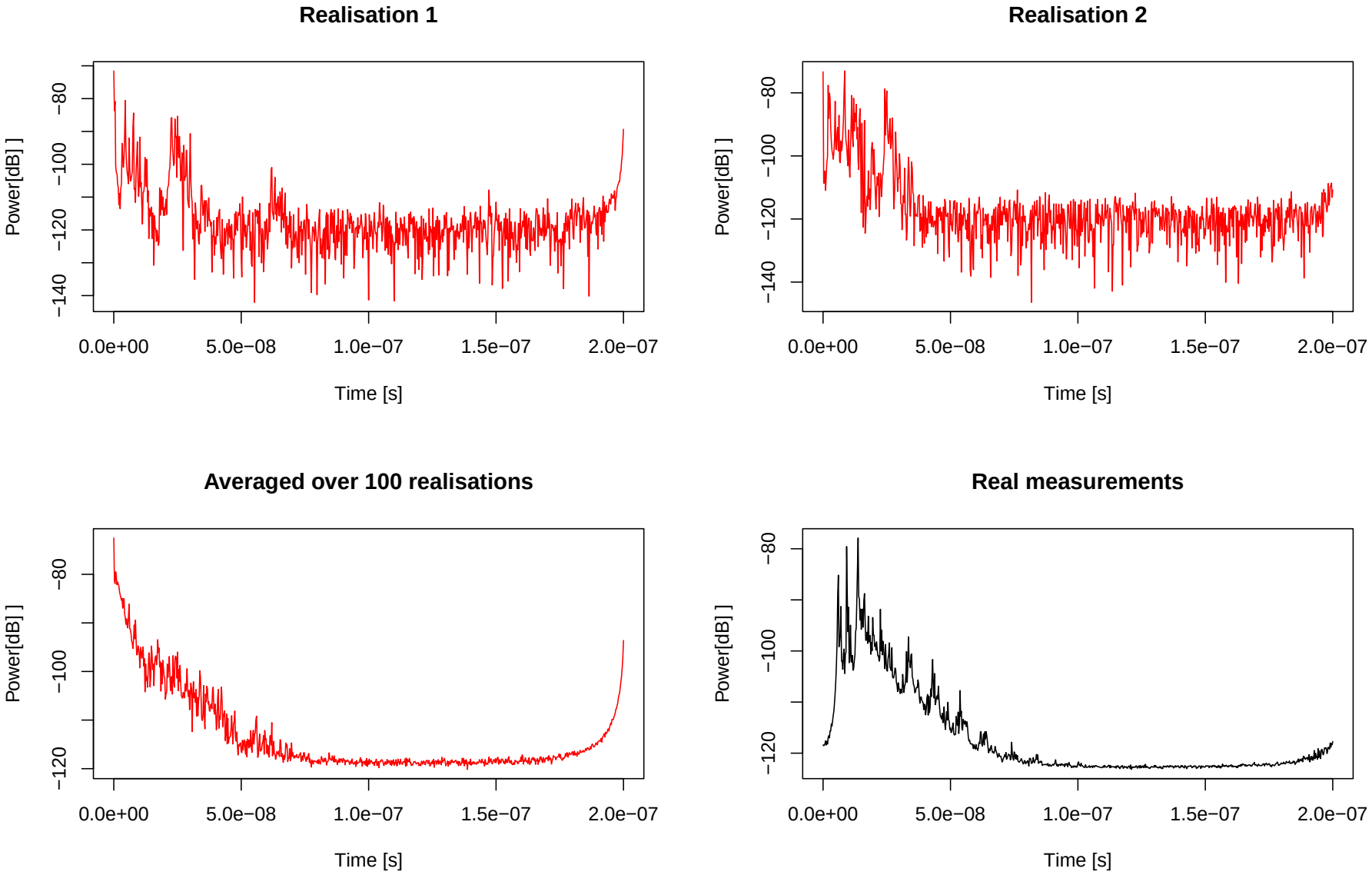


Calibration from measurements

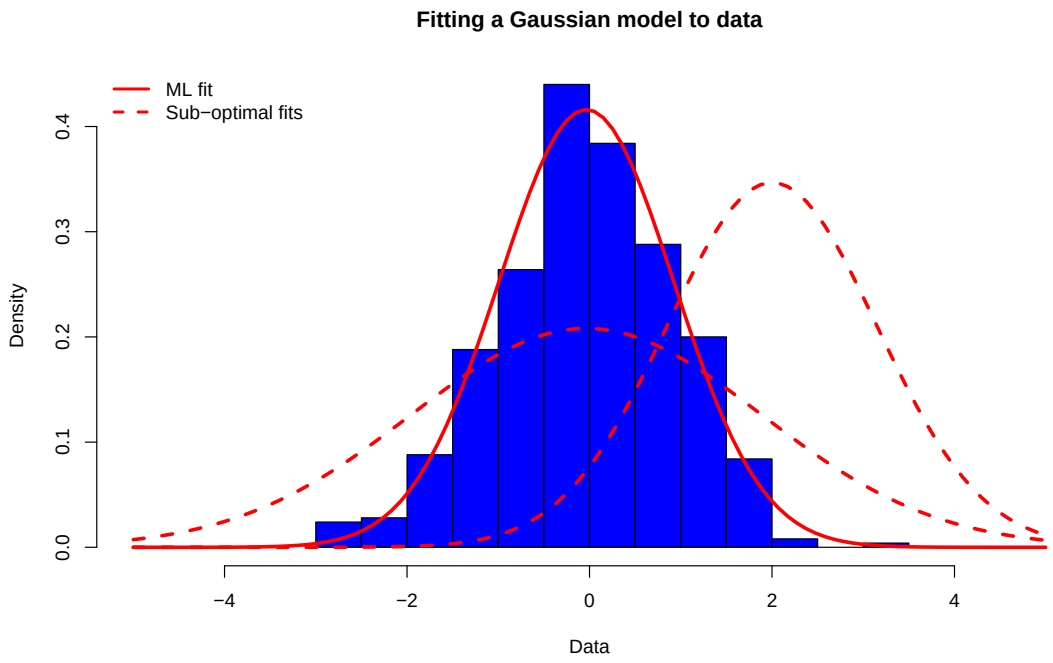
Calibration problem: Given the data $\mathbf{y}_{\text{obs}}$, estimate parameters $\boldsymbol{\theta}$ such that the model $\mathcal{M}_\Theta = \{\mathbb{P}_{\boldsymbol{\theta}} : \boldsymbol{\theta} \in \Theta \subset \mathbb{R}^q\}$ fits to the data.

Classical estimation techniques such as

Maximum Likelihood (ML) estimate: $\hat{\boldsymbol{\theta}}_{\text{ML}} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}\ f(\mathbf{y}_{\text{obs}}|\boldsymbol{\theta})$

Bayesian inference: $p(\boldsymbol{\theta}|\mathbf{y}_{\text{obs}}) \propto f(\mathbf{y}_{\text{obs}}|\boldsymbol{\theta})p(\boldsymbol{\theta})$

require access to the likelihood function $f(\mathbf{y}_{\text{obs}}|\boldsymbol{\theta})$.



Calibrating channel models is challenging

Problem: For most stochastic channel models, the likelihood function $f(\mathbf{y}_{\mathrm{obs}}|\boldsymbol{\theta})$ is intractable and cannot be evaluated numerically.

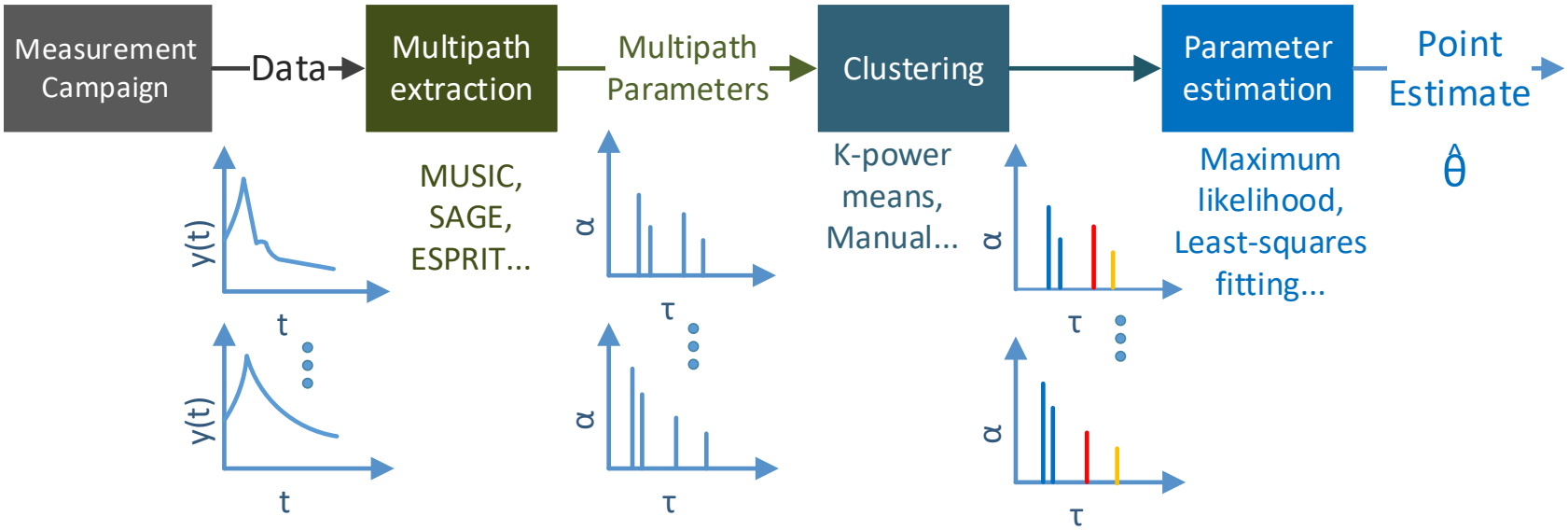
Causes of intractable likelihood:

- The model is simply too complex.
- Variables that are important for model description are unobserved.
- The likelihood function has not been derived yet for a newly constructed model.

Therefore, standard estimation techniques are unrealizable.

State-of-the-art Calibration Method

Multi-step approaches are common:



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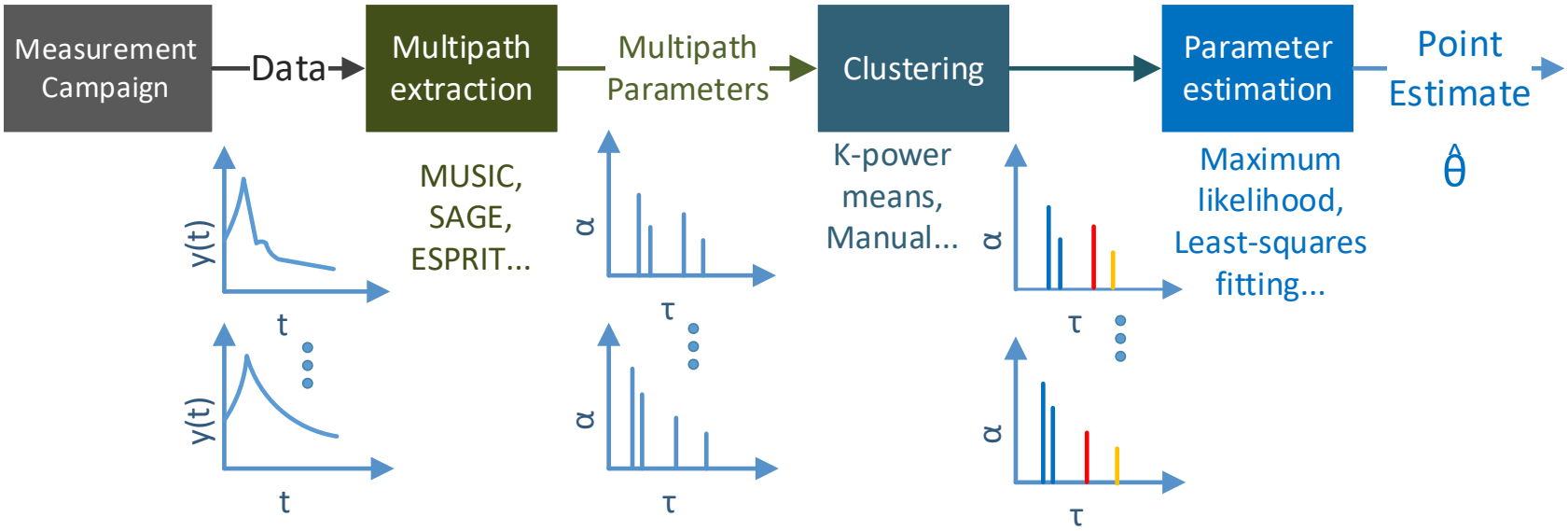

Drawbacks:

- Requires sophisticated algorithms (multipath extraction, clustering) — cumbersome to use due to a number of heuristic choices
- Overall performance of these algorithms are hard to investigate
- Calibration methods are specific to the models
  - redesign for new models
  - hard to compare models due to lack of a common calibration method

Calibration of Radio Channel Models is a LFI problem

Main observation: Calibration of stochastic channel models is a likelihood-free inference (LFI) problem:

- The likelihood function is intractable and cannot be evaluated numerically.
- Easy to simulate data from them.

$\Rightarrow$ Stochastic radio channel models are generative (implicit, simulator-based) models.

Therefore, likelihood-free inference methods such as Approximate Bayesian Computation (ABC) [Sisson, 2018] can potentially be used to calibrate them.

Outline

Approximate Bayesian Computation (ABC)

ABC is a likelihood-free inference method that permits sampling from the (approximate) posterior of a generative model.

Rejection ABC algorithm

- Sample $\boldsymbol{\theta}^* \sim p(\boldsymbol{\theta})$
- Simulate data from model, $\mathbf{y} \sim f(\cdot|\boldsymbol{\theta}^*)$
- If $\rho\left(S(\mathbf{y}), S(\mathbf{y}_{\text{obs}})\right) < \epsilon$, accept $\boldsymbol{\theta}^*$

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- $\rho(\,\cdot\,,\,\cdot\,)$ is a distance metric
- $S(\,\cdot\,)$ is the summarizing function
- ϵ is a tolerance threshold
- Accepted samples $\theta_1, \ldots, \theta_N$ are iid from the approximate posterior:

$$p(\theta|\rho(S(\mathbf{y}_{\text{obs}}), S(\mathbf{y}) < \epsilon) \approx p(\theta|\mathbf{y}_{\text{obs}})$$

Approximate Bayesian Computation (ABC) — contd.

The "approximation" in Bayesian inference arises due to

- use of tolerance threshold in accepting parameter samples
- summarizing the data into a few statistics. If $S(\cdot)$ is a *sufficient statistic* of $\mathbf{y}$, then

$$p(\boldsymbol{\theta}|\rho(S(\mathbf{y}), S(\mathbf{y}^*)) < \epsilon) = p(\boldsymbol{\theta}|\rho(\mathbf{y}, \mathbf{y}^*) < \epsilon)$$

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Ingredients required for implementing an ABC algorithm:

- distance metric $\rho(\cdot, \cdot)$
- summary statistics $S(\cdot)$
- tolerance threshold ϵ

$$\rho(S(\mathbf{y}), S(\mathbf{y}^*)) < \epsilon$$

Approximate Bayesian Computation (ABC) — contd.

Choosing ϵ

- In practice, select ϵ as a small percentile of the simulated distances, i.e. given $\{(\boldsymbol{\theta}_i^*, S(\mathbf{y}_i^*)\}_{i=1}^M$, accept the ϵM samples of $\boldsymbol{\theta}_i^*$ with the least $\rho(S(\mathbf{y}), S(\mathbf{y}_i^*))$

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Choosing $S(\cdot)$

- The choice of statistics is non-trivial as it involves trade-off between
  1. information loss due to summarization
  2. curse of dimensionality
- Domain knowledge is vital!

Approximate Bayesian Computation (ABC) — contd.

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Choose $\rho(\cdot, \cdot)$

- Euclidean distance for summary-based ABC
- Integral probability metrics such as maximum mean discrepancy (MMD), Wasserstein distance

We take ρ to be the MMD.

Outline

Maximum Mean Discrepancy (MMD)

We take ρ to be the MMD, which is a notion of distance on probability distributions or data-sets.

Given a kernel k, a distribution $\mathbb{P}$ can be mapped to a function space $\mathcal{H}_k$ as

$$\mu_{\mathbb{P}}(\,\cdot\,) = \mathbb{E}_{X\sim\mathbb{P}}[k(X,\,\cdot\,)] = \int_{\mathbb{R}^d} k(\mathbf{x},\,\cdot\,)\mathbb{P}(\mathrm{d}\mathbf{x}),$$

where $\mu_{\mathbb{P}}$ is called the kernel mean embedding of $\mathbb{P}$ [Muandet et al., 2017].

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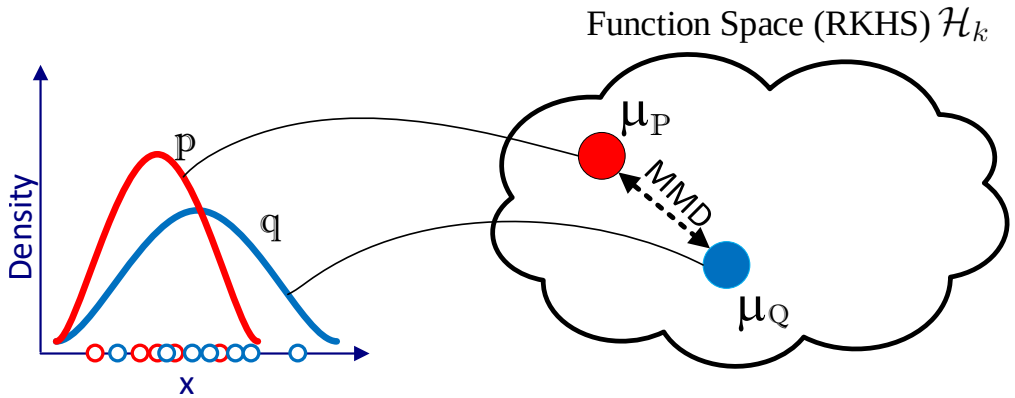
where $\mu_{\mathbb{P}}$ is called the kernel mean embedding of $\mathbb{P}$ [Muandet et al., 2017]. The MMD between $\mathbb{P}$ and $\mathbb{Q}$ is defined as

$$\mathrm{MMD}_k[\mathbb{P},\mathbb{Q}] = \|\mu_{\mathbb{P}} - \mu_{\mathbb{Q}}\|_{\mathcal{H}_k}$$



Remark: The MMD compares infinitely many moments of $\mathbb{P}$ and $\mathbb{Q}$.

Computing MMD from Data-set

The reproducing property of k yields

$$\mathrm{MMD}_k^2[\mathbb{P},\mathbb{Q}] = \mathbb{E}_{X,Y\sim\mathbb{P}}[k(X,Y)] \\ - 2\mathbb{E}_{X\sim\mathbb{P},Y\sim\mathbb{Q}}[k(X,Y)] + \mathbb{E}_{X,Y\sim\mathbb{Q}}[k(X,Y)]$$

Unbiased empirical estimate of $\mathrm{MMD}_k^2[\mathbb{P},\mathbb{Q}]$ from data-sets $\mathbf{X}$ and $\mathbf{Y}$:

$$\widehat{\mathrm{MMD}}_k^2[\mathbf{X},\mathbf{Y}] = \frac{\sum_{i\neq i'} k(\mathbf{x}_i,\mathbf{x}_{i'})}{N_X(N_X-1)} \\ - \frac{2\sum_{j=1}^{N_Y}\sum_{i=1}^{N_X} k(\mathbf{x}_i,\mathbf{y}_j)}{N_Y N_X} + \frac{\sum_{j\neq j'} k(\mathbf{y}_j,\mathbf{y}_{j'})}{N_Y(N_Y-1)}$$

where $\mathbf{X} = \{\mathbf{x}_1,\ldots,\mathbf{x}_{N_X}\} \overset{iid}{\sim} \mathbb{P}$ and $\mathbf{Y} = \{\mathbf{y}_1,\ldots,\mathbf{y}_{N_Y}\} \overset{iid}{\sim} \mathbb{Q}$

Kernel for radio channel measurements

The space of radio channel measurements is high-dimensional, we define the kernel $k_{\mathcal{Y}}$ to avoid the curse of dimensionality.

Hence, we map $\mathbf{y}$ to its first l temporal moments $\mathbf{m} = [m^{(0)}, \ldots, m^{(l-1)}]^\top$,

$$m_i = \int t^i |y(t)|^2 \mathrm{dt}, \quad i = 0, 1, 2, \ldots, l-1,$$

and construct the kernel $k_{\mathcal{Y}}$ as

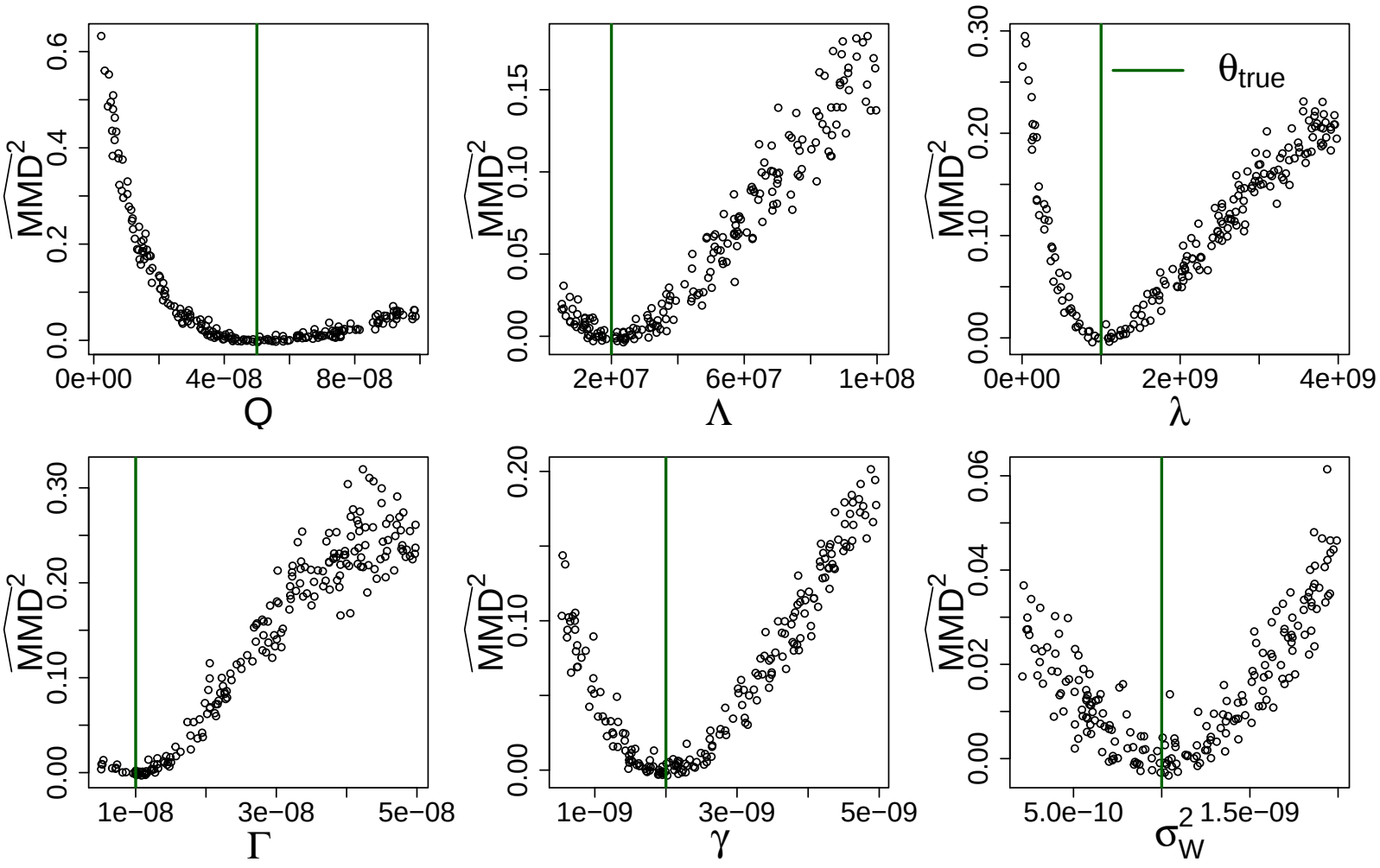
$$k_{\mathcal{Y}}\left(\mathbf{y}, \mathbf{y}'\right) := k_{\mathrm{SE}}\left(\log \mathbf{m}, \log \mathbf{m}'\right), \quad \text{where} \quad k_{\mathrm{SE}}(\mathbf{x}, \mathbf{x}') = \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}'\|_2^2}{l^2}\right)$$

The lengthscale l is set using the median heuristic.

Choice of kernel based on domain knowledge!

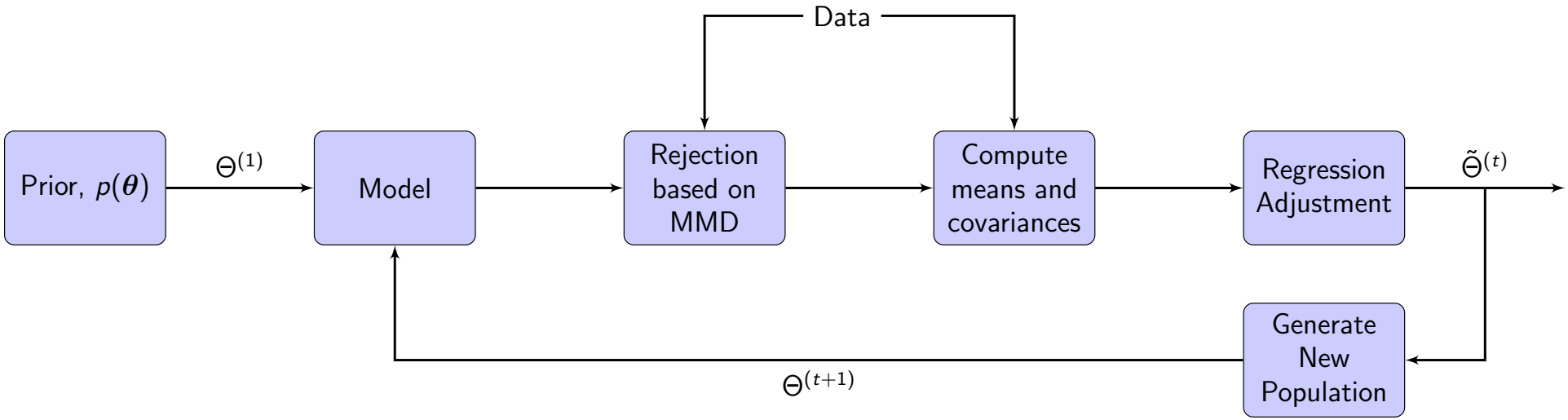
Qualifying the kernel

MMD should be minimum around the true value (green line).



Kernel-based ABC Calibration of Stochastic Channel Models

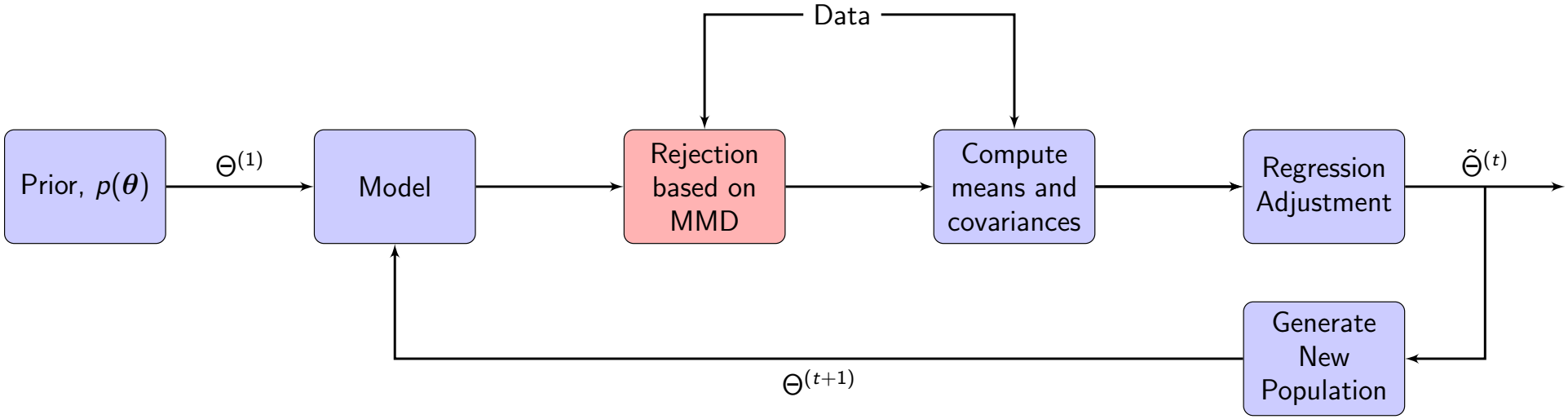
Overview



We use the maximum mean discrepancy (MMD) [Gretton et al., 2012] as the distance metric and combine

- Population Monte Carlo ABC [Beaumont et al., 2009]
- Local-linear regression adjustment [Beaumont et al., 2002]

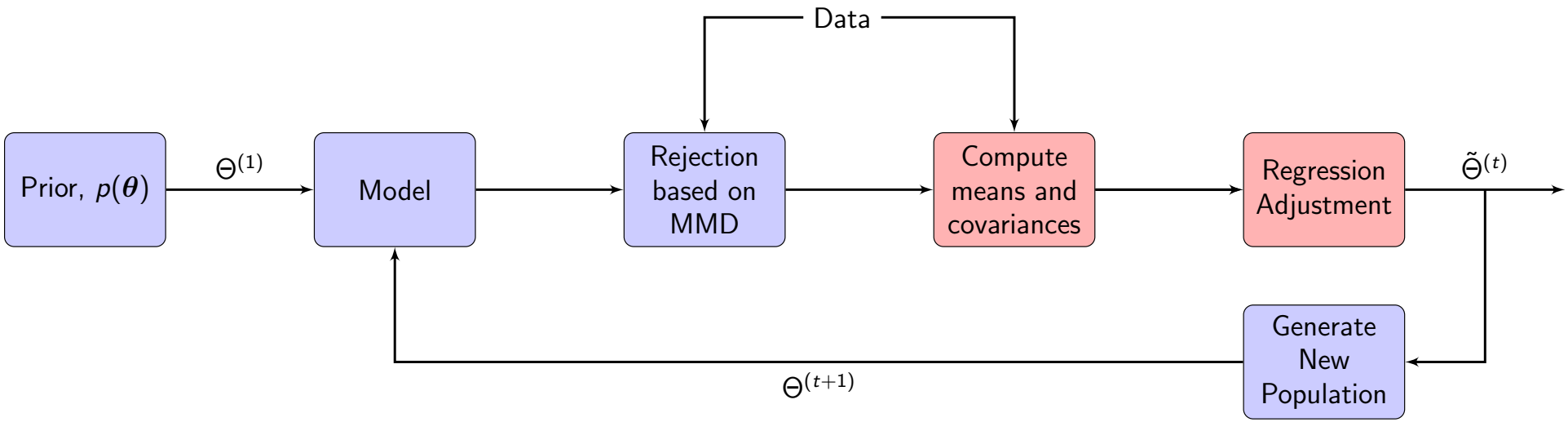
Rejection-ABC based on MMD



Algorithm

- Sample $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_M \sim p(\boldsymbol{\theta})$
- Simulate data from model, $\mathbf{y}_i \sim f(\cdot|\boldsymbol{\theta}_i), i = 1, \ldots, M$
- Compute $\widehat{\mathrm{MMD}}^2_{k_{\mathcal{Y}}}[\mathbf{y}_i, \mathbf{y}_{\mathrm{obs}}]$, $i = 1, \ldots, M$
- Accept $M_\epsilon = \epsilon M$ samples $\{\boldsymbol{\theta}_i^*, \mathbf{y}_i^*\}_{i=1}^{M_\epsilon}$ corresponding to the smallest distances

Regression adjustment ABC

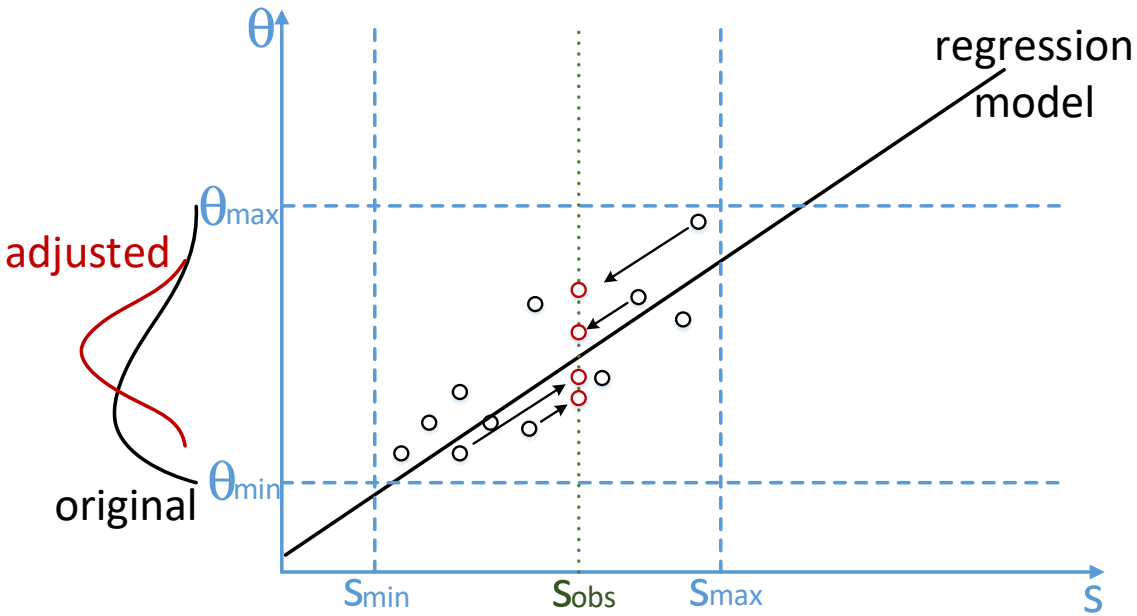


We refine the posterior using regression adjustment.

Regression adjustment ABC

We apply local-linear regression adjustment to the accepted samples to

- weaken the discrepancy between simulated and observed statistics
- to weight the samples according to their MMD distance

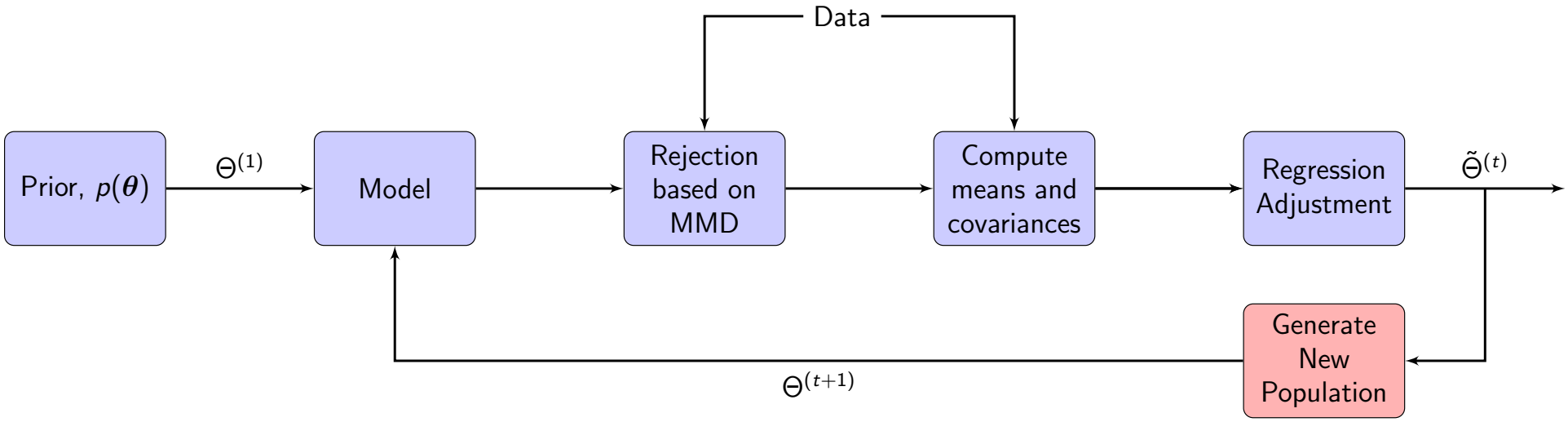


Taking $\mathbf{s}$ to be the *means and covariances of temporal moments*, samples are adjusted as

$$\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i^* - (\mathbf{s}_i - \mathbf{s}_{\text{obs}})^\top \hat{\boldsymbol{\beta}}, \quad i = 1, \ldots, M_\epsilon,$$

where $\hat{\boldsymbol{\beta}}$ is the solution to a weighted least-squares problem.

Population Monte Carlo ABC



Population Monte Carlo ABC

Population Monte Carlo (PMC) ABC is a sequential Monte Carlo method that iteratively improves the posterior.

PMC-ABC with Regression Adjustment

for iteration $t = 1, \ldots, T$

1. Sample $(\boldsymbol{\theta}_1^*, \ldots, \boldsymbol{\theta}_M^*)$ from previous population[a] $\tilde{\Theta}^{(t)}$ with some probability
2. Generate new population $\Theta^{(t+1)} = (\boldsymbol{\theta}_1^{(t+1)}, \ldots, \boldsymbol{\theta}_M^{(t+1)})$ using a proposal density
3. Apply rejection step using MMD on $\Theta^{(t+1)}$ and regression adjustment to get $\tilde{\Theta}^{(t+1)}$

end

[a] For $t = 1$, parameters are sampled from the prior.

Outline

Example 1: Saleh-Valenzuela (S-V) model

Recap

Impulse response:

$$h(t) = \sum_{l} \sum_{p} \beta_{pl} \delta \left(t - \left(T_l + \tau_{pl} \right) \right)$$

Stochastic Model:

- $T_l \sim \mathrm{PoissonPP}(\mathbb{R}^+, \Lambda)$
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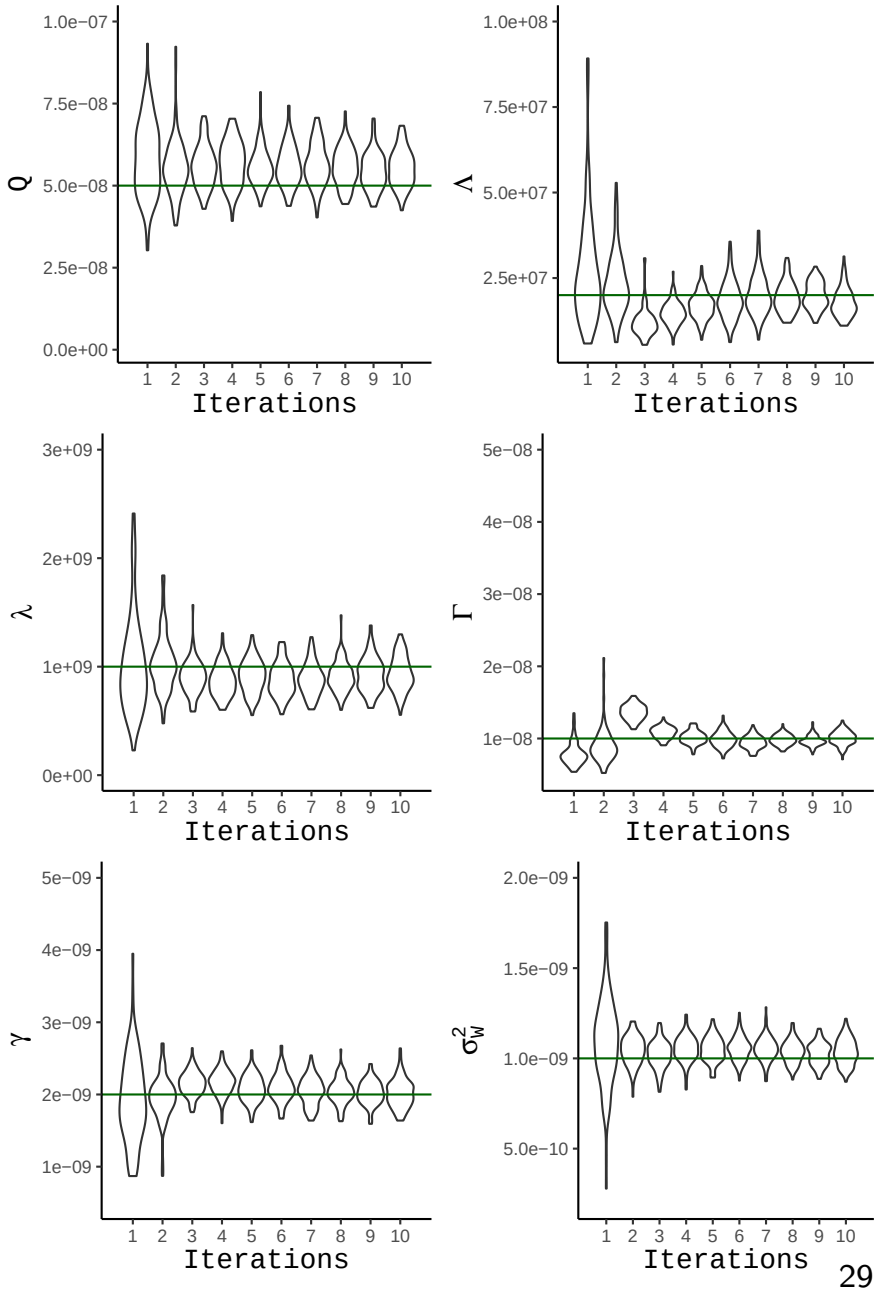
Parameters to be calibrated: $\boldsymbol{\theta} = [Q, \Lambda, \lambda, \Gamma, \gamma, \sigma_W^2]^\top$

Simulation experiment (S-V model)

Settings: Flat priors, $I = 4$, $M = 2000$, $M_\epsilon = 100$, $N_{\text{sim}} = 100$, $N_{\text{obs}} = 1000$

Observations:

- Approximate posteriors concentrate around the true value.
- The algorithm converges rather quickly and the posteriors taper as the iterations proceed.
- Even the first iteration gives a reasonable estimate.



Application to Measured Data

We apply the kernel-based ABC method on real measurements[1].

- Environment: small conference room of dimension $3m \times 4m \times 3m$
- Frequency range: 58 GHz to 62 GHz, 801 frequency samples
- 5×5 virtual planar array at both sides, leading to 625 channel realizations

[1] From [Gustafson et al., 2016]

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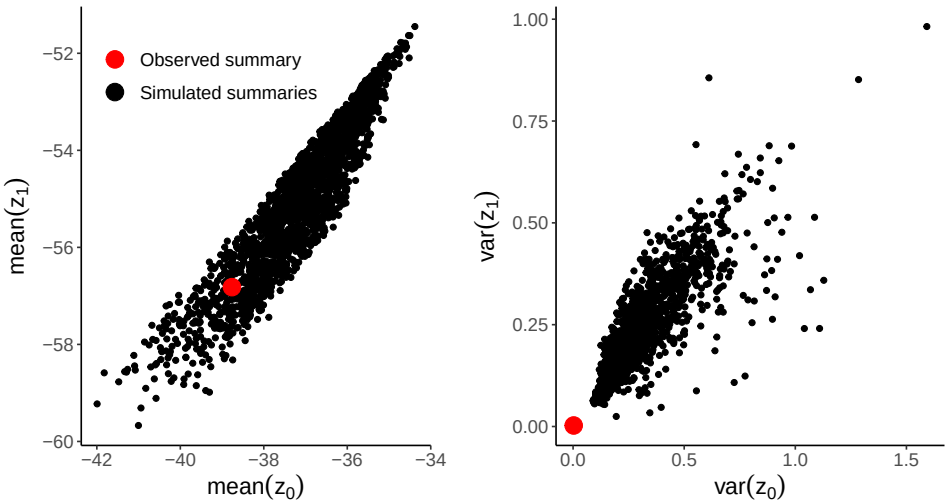

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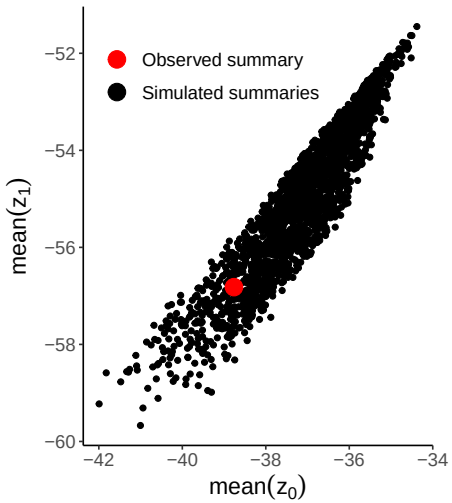

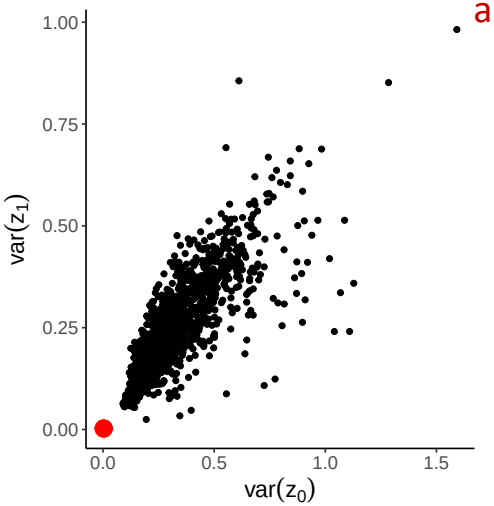

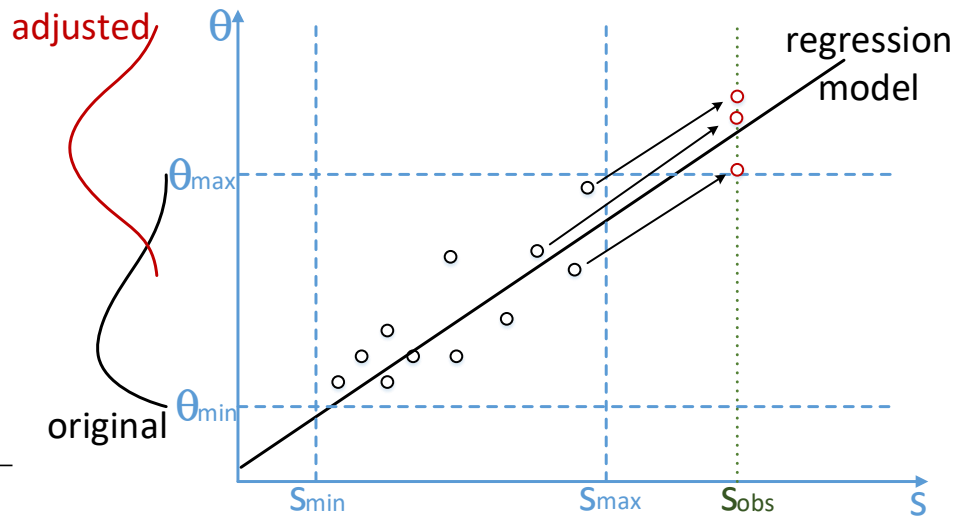

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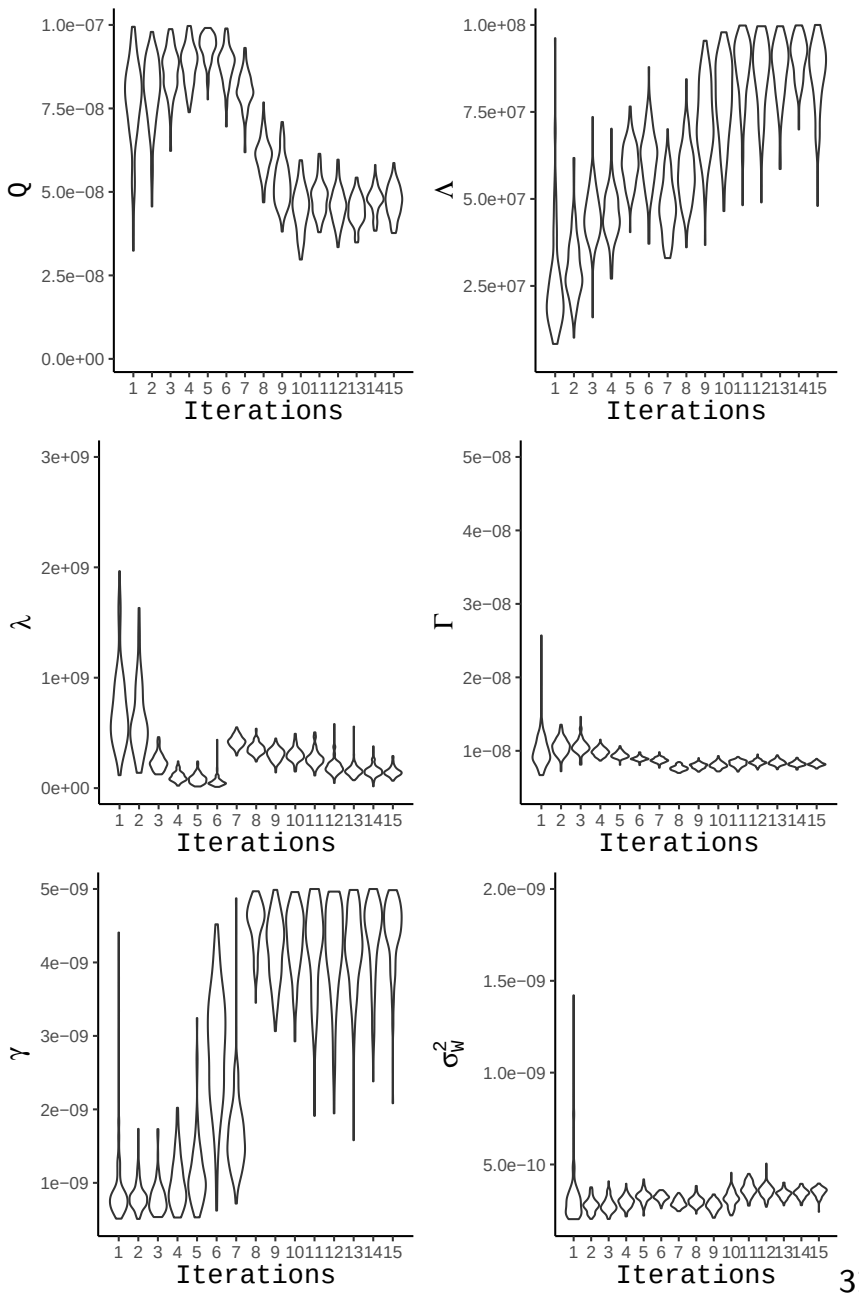
Application to Measured Data (S-V model)

Observations:

- The marginal approximate posteriors for λ, Γ, and σ_W^2 are highly concentrated.
- Posteriors for Γ and σ_W^2 appear to converge from the second iteration itself, indicating that these parameters affect the MMD the most.
- Posteriors for Q, Λ and γ take around eight or nine iterations to converge to a different location in the prior range than where they began from, unlike the simulation experiment.

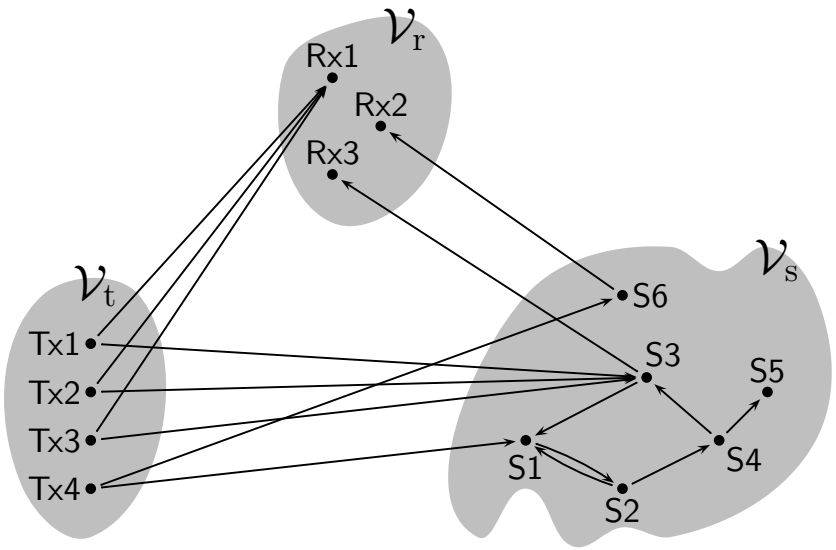


Example 2: Propagation Graph (PG) model

We consider a simple directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$.

Vertex set $\mathcal{V}$: The transmitters, receivers, and scatterers are represented by vertices in the set: $\mathcal{V} = \mathcal{V}_{\mathrm{t}} \cup \mathcal{V}_{\mathrm{r}} \cup \mathcal{V}_{\mathrm{s}}$.

Edge set $\mathcal{E}$: Wave propagation between the vertices is modeled by edges in $\mathcal{E}$. If wave propagation from $v \in \mathcal{V}$ to $v' \in \mathcal{V}$ is possible, then $(v, v') \in \mathcal{E}$.



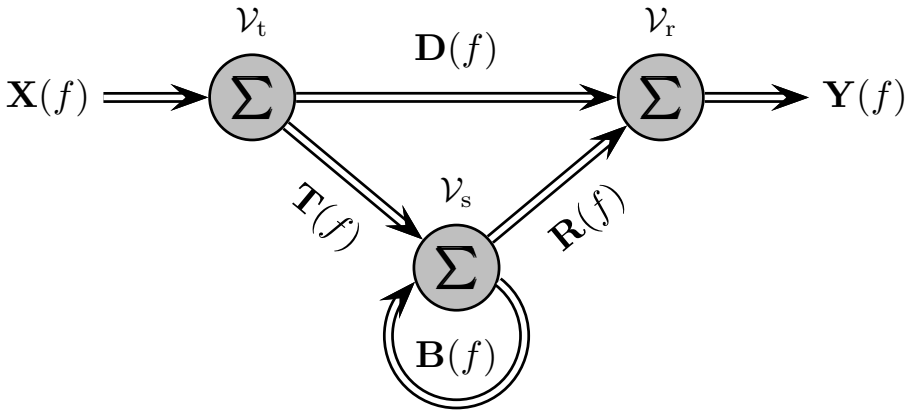
Rules for propagation:

- The sum of signals impinging via the incoming edges of a scatterer are re-emitted via the outgoing edges.
- An edge $e = (v, v') \in \mathcal{E}$ transfers the signal from v to v' according to an edge transfer function $A_e(f)$.

Transfer Matrix

PG model

Edge transfer functions can be grouped into matrices according to the vector signal flow graph:



The transfer matrix is obtained as:

$$\begin{aligned}\mathbf{H}(f) &= \mathbf{D}(f) + \mathbf{R}(f)(\mathbf{I} + \mathbf{B}(f) + \mathbf{B}(f)^2 + \mathbf{B}(f)^3 + \dots)\mathbf{T}(f) \\ &= \mathbf{D}(f) + \mathbf{R}(f)(\mathbf{I} - \mathbf{B}(f))^{-1}\mathbf{T}(f), \qquad \rho(\mathbf{B}(f)) < 1.\end{aligned}$$

Stochastic Propagation Graph

Edge transfer function for edge $e = (v, v') \in \mathcal{E}$

$$A_e(f) = g_e(f) \exp[j(\psi_e - 2\pi\tau_e f)]$$

where

- $\psi_e \sim \mathcal{U}(0, 2\pi)$ is the phase
- $\tau_e = \|(\mathbf{r}_w - \mathbf{r}_v)\|/c$ is the propagation delay between vertex w and v with position vectors $\mathbf{r}_w$ and $\mathbf{r}_v$, respectively, randomly generated
- $g < 1$ is a gain constant,
- c is the speed of light in vacuum.

The edge gain, $g_e(f)$ is calculated as:

$$g_e(f) = \begin{cases} \frac{1}{(4\pi f \tau_e)}; & e \in \mathcal{E}_{\mathrm{d}} \\ \frac{1}{\sqrt{4\pi\tau_e^2 f \mu(\mathcal{E}_{\mathrm{t}})\mathcal{S}(\mathcal{E}_{\mathrm{t}})}}; & e \in \mathcal{E}_{\mathrm{t}} \\ \frac{g}{\mathrm{odi}(e)}; & e \in \mathcal{E}_{\mathrm{s}} \\ \frac{1}{\sqrt{4\pi\tau_e^2 f \mu(\mathcal{E}_{\mathrm{r}})\mathcal{S}(\mathcal{E}_{\mathrm{r}})}}; & e \in \mathcal{E}_{\mathrm{r}} \end{cases}$$

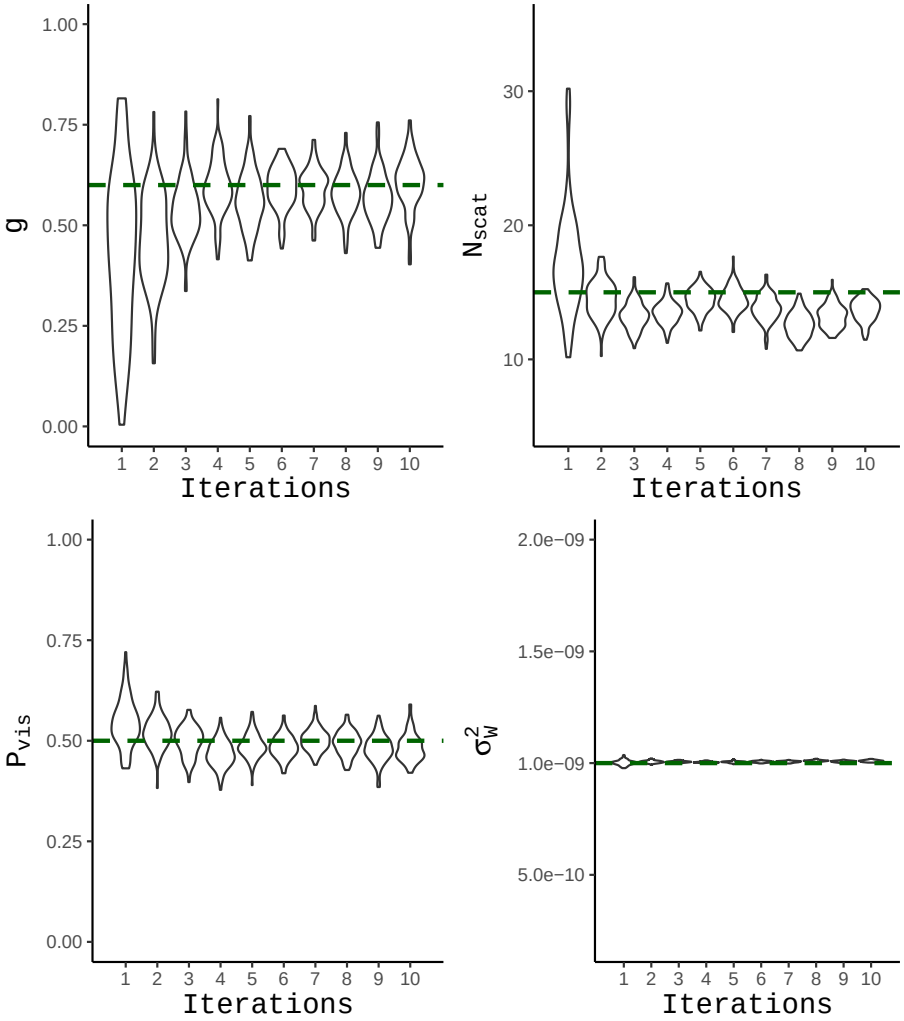
Parameters to calibrate:
$\boldsymbol{\theta} = [g, N_s, P_{\mathrm{vis}}, \sigma_W^2]^\top$

$$\mu(\mathcal{E}_{\mathrm{a}}) = \frac{1}{|\mathcal{E}_{\mathrm{a}}|} \sum_{e \subset \mathcal{E}_{\mathrm{a}}} \tau_e, \quad \mathcal{S}(\mathcal{E}_{\mathrm{a}}) = \sum_{e \subset \mathcal{E}_{\mathrm{a}}} \tau_e^{-2}, \quad \mathcal{E}_{\mathrm{a}} \subset \mathcal{E},$$

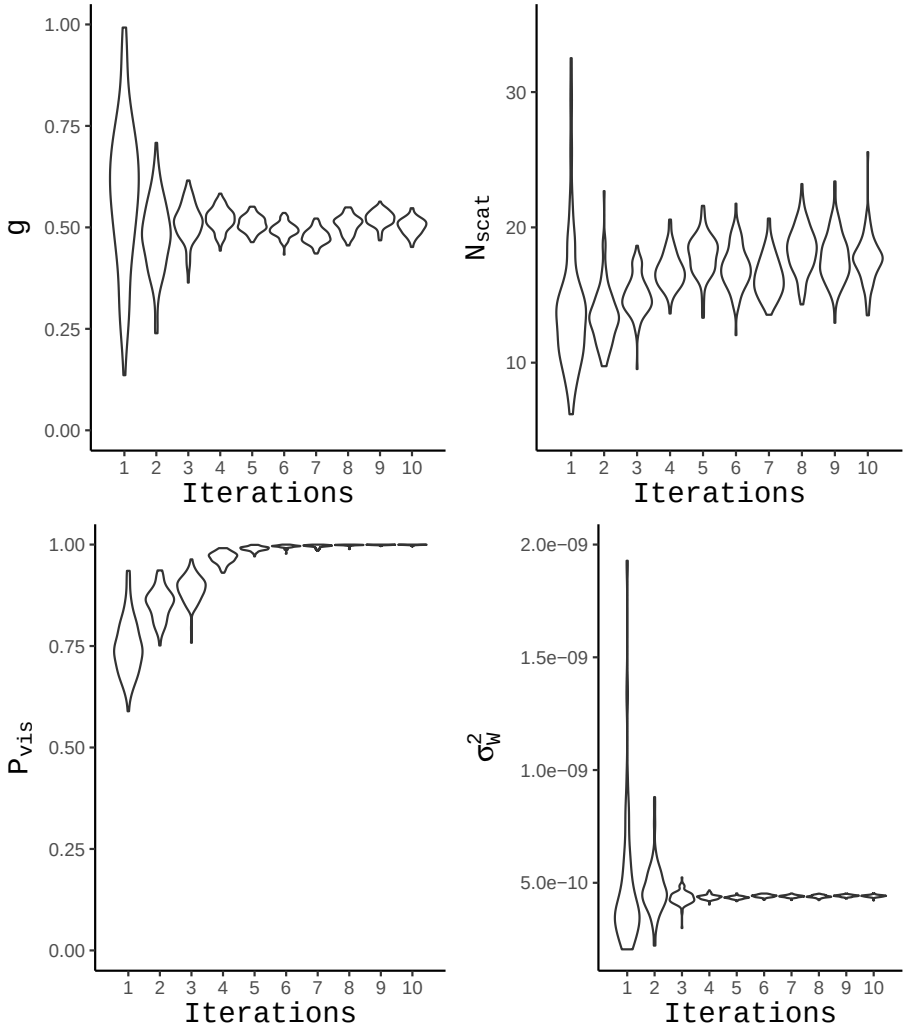
Example 2: Propagation Graph (PG) model

We also apply the proposed algorithm on the Propagation Graph (PG) model which is of a different mathematical structure than the SV model:

Simulated Data:



Measured data:

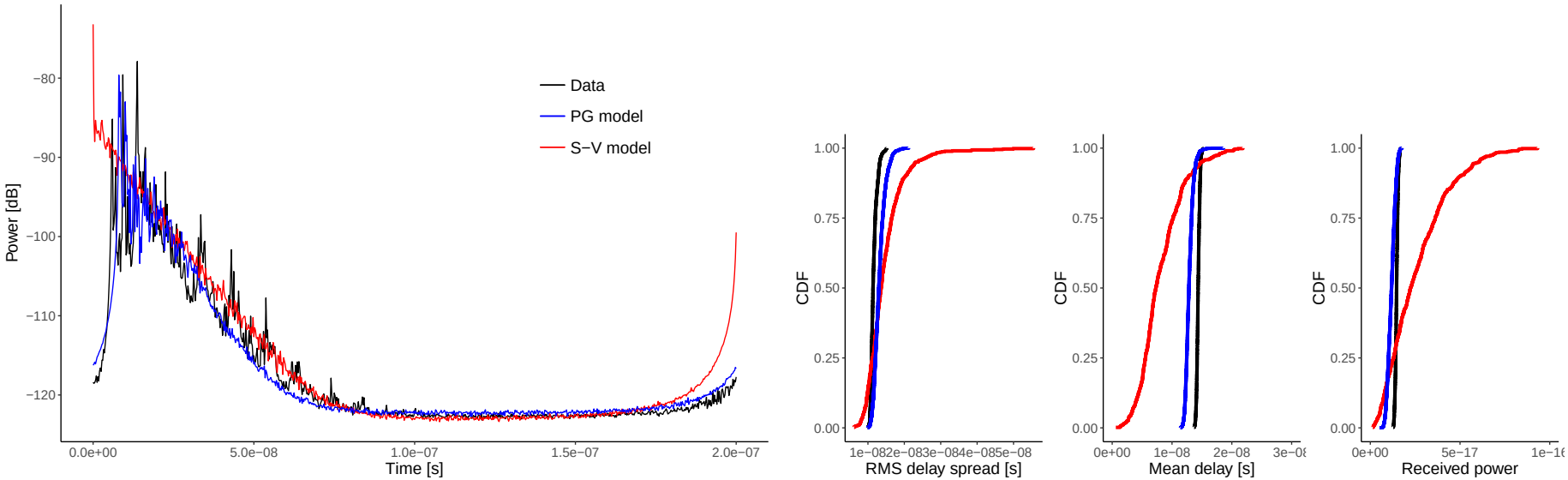


Observations (PG model)

- The algorithm converges quickly.
- Approximate posteriors are highly concentrated around the true values.
- The approximate posterior for g starts off very wide and then gets narrower as the iterations proceed.
- The proposed method is able to accurately calibrate the PG model.
- Results on measured data is similar to what is observed in the simulation experiment.
- Convergence is achieved after around four or five iterations.

Model Fit Comparison

Comparing the two model fits in terms of the averaged power delay profile (APDP), rms delay spread, mean delay, and received power:

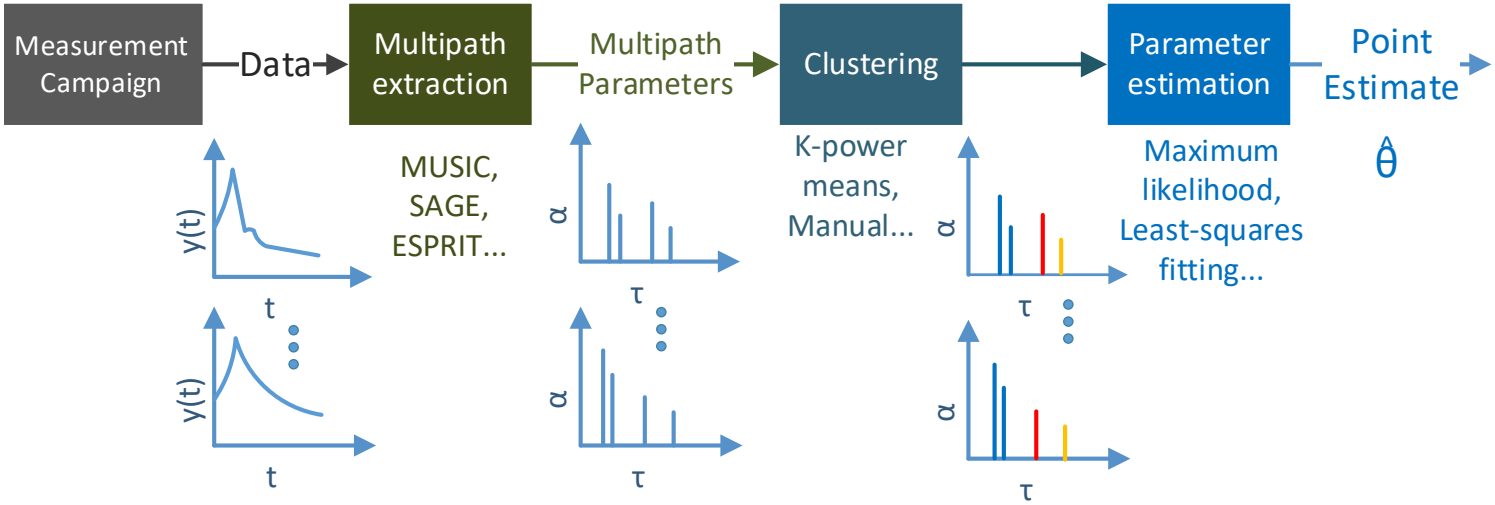


- Both S-V and PG models are able to fit the APDP well.
- S-V model is not able to replicate the initial peaks in the data, while the PG model represents them well.
- The PG model captures the behavior of the empirical cdfs well, while S-V model fails to do so.

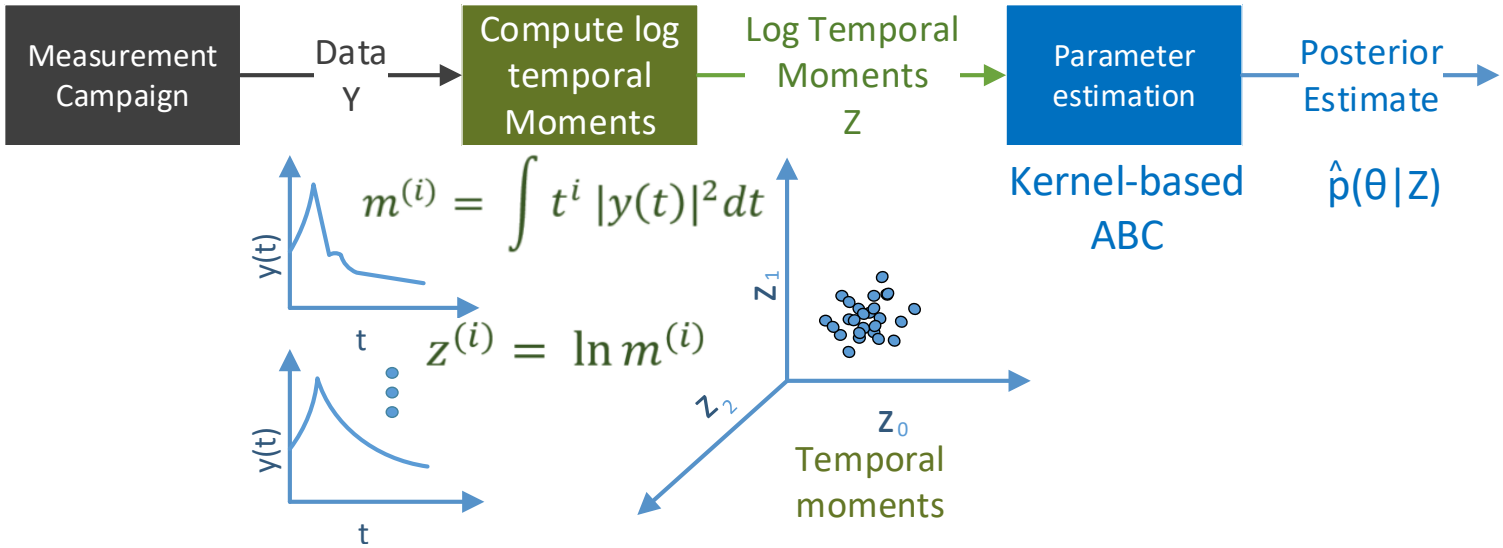
Outline

Take-home message

State-of-the-art calibration method:



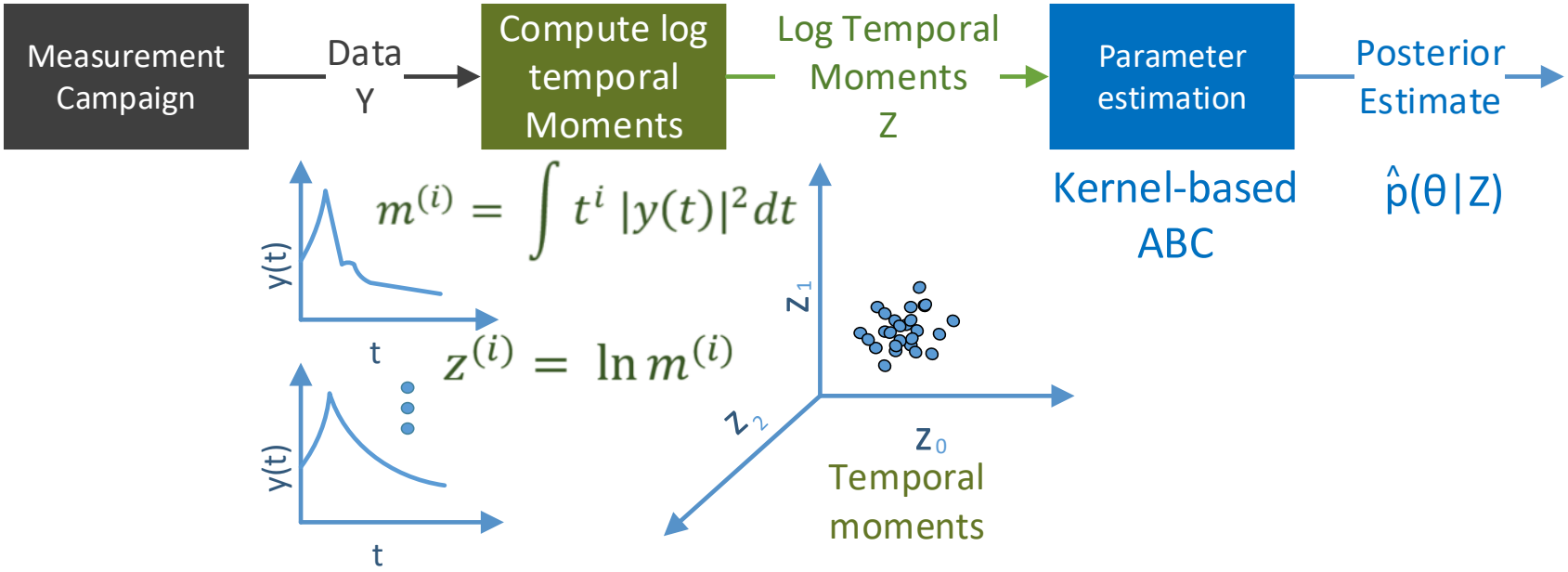
Proposed method using easy-to-compute general summaries:



Take-home message

Advantages:

- General calibration method applicable to different channel models
- Simpler processing chain
- Fit is based on explicit choice of summaries
- Information on posterior is obtained (not only point estimates)



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